

Age Versus Performance for a Select Group of Nordic Master's Skiers

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ABSTRACT

The purpose of this study is to explore a technique for evaluating the longitudinal aging characteristics for a small, select group of master's nordic ski racers. The study gathers race times from the public domain and reduces them to race pace in min/km, regardless of the length of the race or other environmental variables. It is shown that the time evolution of race pace for individual skiers in the sample indicates clear trends over two decades. The average rate of change for the sample of 30 is $+0.1 \pm 0.1$ percent/year, a slight decline in performance versus age. Strikingly, one-third of the population actually shows improved performance during this time. The results are compared to other studies of aging in skiing and other sports.

Introduction

The effects of aging on older athletes has been the subject of studies that have attempted to generalize results to the expected decline in performance with advancing age. In the sports of swimming and track and field, race lengths and conditions, apart from some weather-related variations and differences in venue altitude, allow researchers to compare athletes from many different events in order to build numbers and increase the statistical significance of the results. For example, in their cross-sectional study Rahe and Arthur (1975) used swim data from United States Master's Swimming meets to compute how times slowed among 5-year age groups between 25-29 and 55-59. They concluded that there was a general slowing at the rate of about 0.8% per year within these age groups. More recently Rubin and Rahe (2010) have revisited this subject with a much larger database and found a linear decline in performance of 0.6% per year up to age 70 and an exponential decrease beyond that. In a study restricted to longer-distance events, Bongard et al. (2007), argued that the decline in performance was best fit by a quadratic form. Their model predicts a roughly 50% loss in performance between ages 15 and 95, an average of 0.6% per year overall, but slower in the younger age groups and faster in the older age groups owing to the quadratic nature of the relation between speed and age.

In a review of several sports, including swimming, Baker & Tang (2010) examined master's world records using both linear and curvilinear models in their study. Staying with swimming, their results (using the linear analysis and tabulated in their Table 3) showed a decline for men of slightly less than 0.8% per year between ages 35 and 85 and for women slightly more than 0.8%. Thus, these cross-sectional studies taken together seem to show a rate of decline between 0.6% and 0.8% per year between young adulthood and at least 85.

The difficulty with cross-sectional comparisons is that they do not follow a particular age cohort as it ages. Rather they compare the current 45-49 year-old athlete with the current 80-84 year-old athlete and so on. This has the built in problem that athlete cohorts separated by decades will have been tutored in different training methods and different ways of practicing the sport at its highest level. Top level athletes will often try to adapt to the newer methods of technique and training but perfect propagation of change may not occur across several decades. Making corrections for these problems is difficult and these points have been noted by Baker & Tang (2010). Furthermore, these are statistical comparisons and an individual athlete may not be expected to conform to the model developed. In fact, each individual athlete may wish to determine how he/she will change with time. For the moment, these statistical models are the best tool available. On the other hand, as Baker & Tang (2010) point out, the longitudinal behavior of particular aging athletes carries with it the unknowns of changing "health, motivation and environmental conditions." There are also no easy algorithms to correct for these effects. There have been studies of longitudinal behavior of master's athletes in a number of sports. Starkes, Weir & Young (2003) have reviewed both cross-sectional and longitudinal studies in running and swimming and conclude that both kinds of populations show similar declines in performance with advancing age with longitudinal decline occurring at a somewhat slower rate.

Nordic skiing comparisons are even more difficult since they have the additional problems of errors in course length, variations in race distances, differences in altitude

among race venues, differences in terrain (whether courses are primarily flat or hilly or a mixture of both) and extreme weather condition changes which may affect the choice of ski and wax (and the ability of the skier to make the correct choice). Possibly there are other factors as well. Heil (2010), in a cross-sectional study of the Boulder Mountain Tour ski race results (a point-to-point 32 km race in Idaho), has developed an “age-adjusted” correction that allows him to compare older skiers with younger ones. He then applies a correction based on this average to individual skiers to correct their times downward in age. The analysis was carried out for the race results for the 2011 Boulder Mountain Tour (Heil 2011). As he points out, those exceptional older athletes with times much faster than their age-class averages move up very high in the adjusted rankings, bettering actual race winners in some cases. This procedure has also been used for running races by USATF (2013).

Method

I was motivated to try to develop a longitudinal analysis and apply it to a small, select group of master's Nordic skiers. All are known to me through personal observation and, although no extensive interviews have been conducted, none of the skiers have any obvious debilitating long-term health issues. These are, of course, qualitative and in some cases, anecdotal judgments. Nevertheless, I wanted to investigate a criterion for evaluating each individual as a function of time to see, for example, how each compared to the statistical performance-versus-aging determinations referenced above of 0.6-0.8% per year loss of performance. All the athletes in the sample are serious competitors and engage in year-round activity in addition to Nordic skiing. Many of the sample do very well in races, some are elite athletes (Olympians and former National Team or high-level college skiers) but many of the sample don't do particularly well (that is, they are average citizen racers). The characteristics of the 30 skiers chosen are summarized in Table 1, where the skiers have been identified only by gender and whether or not they are considered to be elite (E) or citizen (C) racers. Also included in the table are the span in years for the races considered (ΔT) and the number of races used in the sample. The age of the skiers at the start of the study has not been tabulated, in order that skiers may not identify themselves in the table, but are known to me and will be used in the discussion. Those starting ages range from 19 to 64 with the mean age at the start of the study being 39 ± 12 years. The mean number of years spanned for an individual skier is 21 ± 2 and the mean number of races considered is 34 ± 10 . Thus the study encompasses, on the average a longitudinal profile from age 39 to age 60. Twenty of the skiers are classified as citizen and 10 as elite; note that among the citizen classification there are Olympians and college athletes in sports other than skiing. There are 13 female skiers and 17 male skiers among the sample.

As a test criterion for determining the effects of aging on master's Nordic skiers, I have taken data from a large and varied set of skate-technique races dating back to the middle 1980s, including Salomon Series races in Colorado, the Snow Mountain Stampede, the Colorado Governor's Cup, AXCS National Master's races, the American Birkebeiner, the West Yellowstone Rendezvous, the Boulder Mountain Tour, the Master's World Cup, the Olympics and World Cup and a number of other races with results available to me from web searches or in paper form. All data are available in the public domain either as tabulated results on web sites or as paper copy handouts from the races concerned. The data are restricted to skating races. For the purpose of this paper, I have ignored the concerns raised in the introduction

and simply reduced the race data to the skier's race pace, in units of min/km, regardless of race length, for lengths from about 3 km up to 90 km. No sprint races have been included because I expect them to use different physiological energy systems than the longer races. That is, I wanted to be certain that the races were well within the aerobic phase of production of ATP used in muscle contractions (see Enger, Ross & Bailey 2011, for example). In any case, the availability of races much shorter than 5 km is very limited for master's skiers.

With the race data for each skier reduced to pace, the results were plotted against the date of the race. These were done for all 30 skiers in the sample. A few of the results are shown for a selection of 4 skiers from the sample in Figs. 1 to 4. It is clear that the graphs exhibit a large degree of scatter, as expected from the above arguments. In spite of this scatter, significant trends exist among the sample of skiers. *That is, the criterion of skier pace versus date, regardless of race length and other factors seems to yield a meaningful analysis tool for the long-term performance of individual skiers.*

The trends appeared to be approximately linear in nearly all cases and I have fitted a linear relation of the form $P = A + BD$, where P is the pace in min/km, D is the elapsed time since the first race and A , B are constants to be determined. The fits have been carried out using a simple least squares fit (linear regression), assuming each data point has unit weight (see Weisstein 2013, for example) and the resulting models are shown as solid lines in the figures. I have not tried to refine the fits with higher-order terms because visual inspection suggested that the linear fits are adequate, with perhaps two exceptions, to be discussed later. The linear fits will suffice for the purpose of this paper since generalities rather than detailed aging profiles are the objective. In a few cases the data showed apparent outliers that, owing to the relatively small number of data points, slightly skewed the resultant fits. That is, the fits may not be entirely "robust" for those cases; I have made no attempt to correct for these outliers (see Chapter 14 in Press *et al.* 1987 for a discussion of robust fits). In one case a skier had short-term health issues over three separate seasons and the data have been removed for the affected races. *No other data editing has been performed.*

There are several noteworthy properties to the set of graphs. First, some skiers show an obvious decline in speed with advancing age as expected from the discussion in the introduction. But, not all skiers fall into this set. Some skiers show approximately no change with advancing age while still others *actually improve with age*. For the purpose of further discussion, I have summarized the graphical data for all 30 skiers in Table 1, where I have tabulated the slope of the straight-line fit to the data (B) divided by the average value of the pace ($\langle P \rangle$) over the span of the data, along with the standard error in the ratio (σ). These are expressed as percentage change in performance per year and are listed in column 6 of the table. Note that a positive slope means that the skier's performance degrades each year while a negative slope means that the skier actually improves year-over-year. For example, skier number 6 has a slope of +0.42 which means that his pace increases at the rate of 0.42% each year whereas skier number 10 has a slope of -1.06 which means that her performance has improved by an astounding 23% over the course of the study.

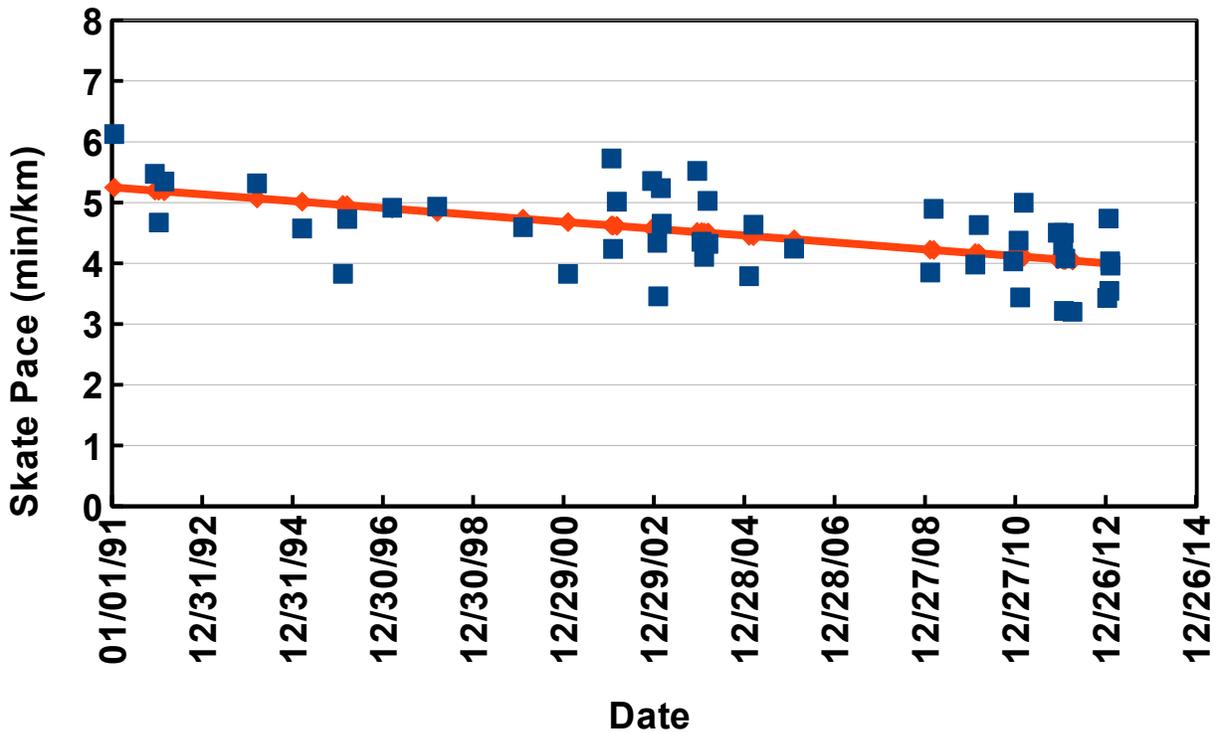


Figure 1. Pace data for skier number 12, a citizen racer whose pace decreases with advancing age at the rate of -1.3 percent/year. The solid line is the result of a linear least-squares fit to the data.

The first two graphs shown (Figs 1 and 2) illustrate the pace data for two citizen racers. One may see that skier number 12 improves with age at the rate of -1.3 ± 0.3 percent/year while skier number 15 loses performance with advancing age at the rate of $+0.7 \pm 0.4$ percent/year. The difference in the slopes of the two lines is 2.0 ± 0.5 percent/year. This is a strong, 4-sigma result and implies that the differences shown are significant ($p < 0.0001$) for a normal distribution of errors (Bevington and Robinson 2003). In head-to-head competitions, then one would expect that skier 15 would have won most races in the early 1990s but that the reverse would be true after about 2008. Of course, the scatter envelope for both skiers is sufficiently large that good races by one skier coupled with bad races by the other would result in occasional reversals of that outcome.

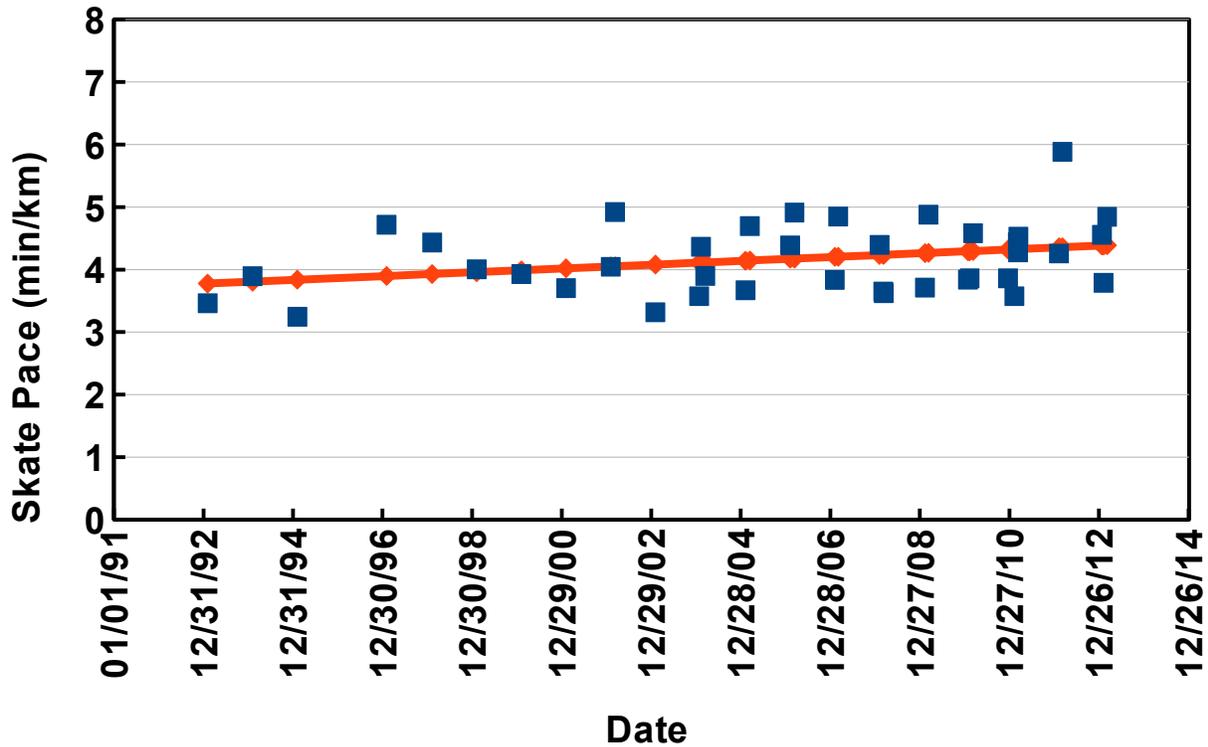


Figure 2. Pace data for citizen skier number 15, whose pace increases over time at the rate of +0.7 percent/year. The solid line is a linear least-squares fit to the data.

In Figs 3 and 4 I have shown data for two elite ski racers, skiers number 22 in Fig 3 and number 20 in Fig 4. Note that the difference in pace versus age is much tighter in Figs 3 and 4 than in Figs 1 and 2. With two exceptions, the slope of the pace for elites stays within the range ± 0.4 percent/year change. The two exceptions (skiers number 24 and 27) are quite large positive slopes (that is, degrading performance), consistent with the largest of the slopes observed for the citizen skiers. It is possible that changing motivation plays an important role in these two cases.

Skier number 22 shows the largest rate of improvement among the elite skiers (-0.4 ± 0.3 percent/year) while number 20 shows a modest rate of decline with age ($+0.2 \pm 0.3$ percent/year). The difference between the two slopes is 0.6 ± 0.4 percent/year which is a 1.5-sigma result. That is, the statistical significance of the difference between these two skiers is slightly above the 87% confidence level ($p \approx 0.13$).

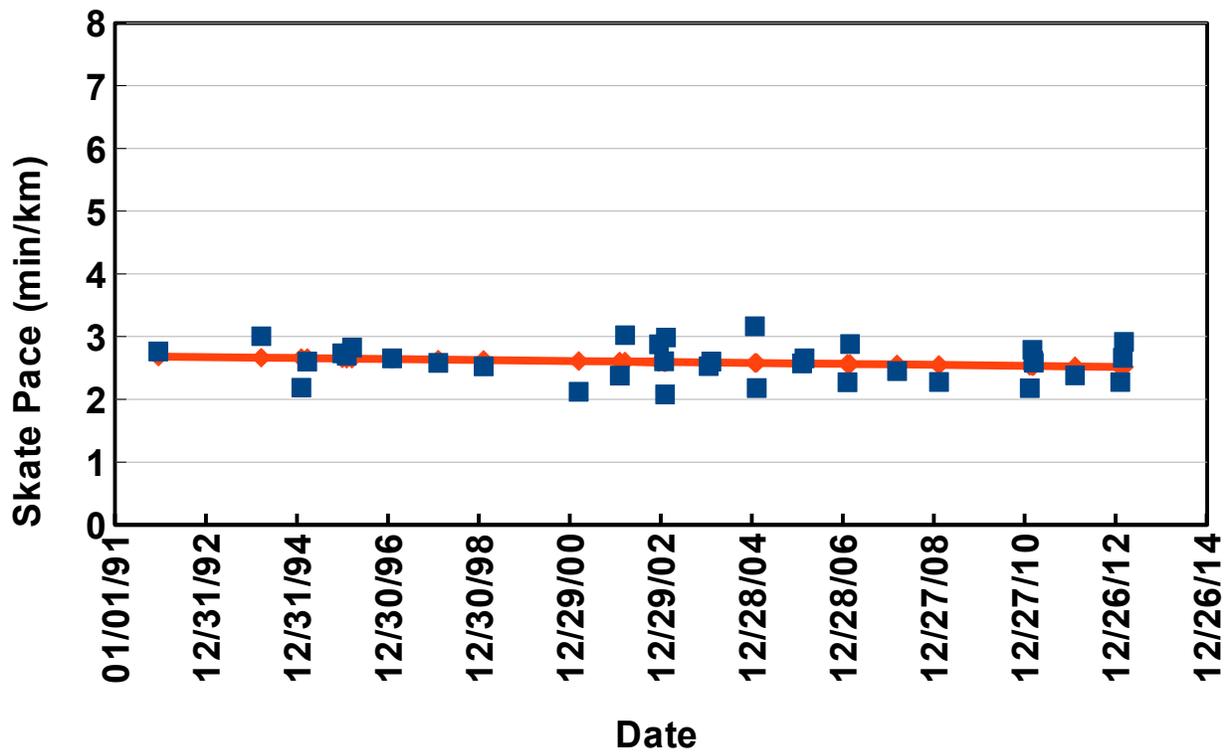


Figure 3. Pace data for elite skier number 22. The least-squares fit to the data yield an improvement in pace at the rate of -0.4 percent/year.

Regarding skiers number 24 and 27, the data almost certainly are not well-fitted by a linear model. This is not so obvious from the errors in the slopes of the linear fits to the data, given in Table 1, but is more apparent from inspecting the graphical data. In both cases, the scatter is small enough to see possible higher-order trends to the data. Skier number 27 is particularly interesting. In her case the pace is flat (no change) from about 1991 until about 2000. Then the pace decreased slightly (that is, the skier improved) until about 2006. Past 2006 there began a rather steep increase in slope. For skier number 24 there appears to be a flat pace followed by a steeper decline. As I mentioned above, it is possible that motivation for continuous intense training may have changed for these two skiers although both still participate in races.

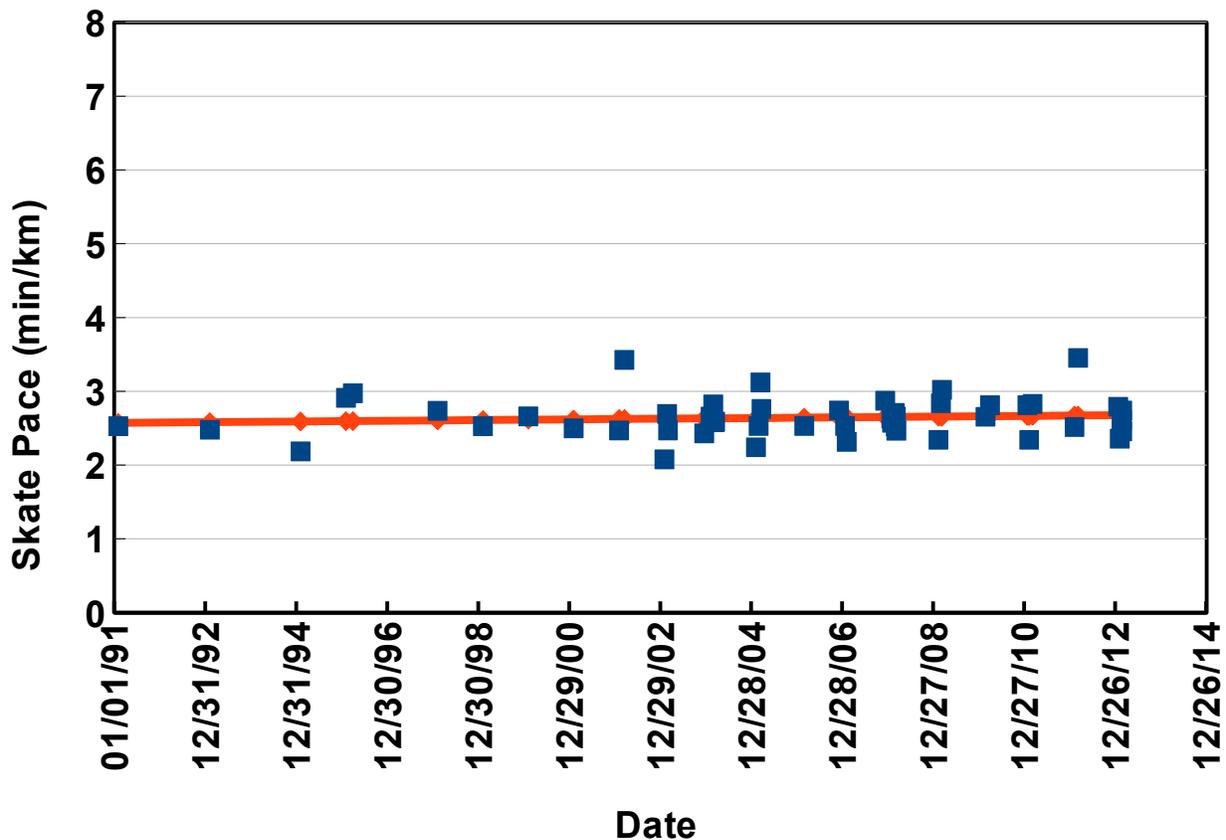


Figure 4. Pace data for elite skier number 20. The linear least-squares fit to the data yield a slope of +0.2 percent/year, indicating a slight increase in pace with advancing age.

Discussion

A. Some Generalities

There are some general conclusions that derive from this study. First, there are meaningful long-term trends evident for every skier in the survey in spite of the scatter caused by variations in race parameters and environmental factors. These trends vary from skier to skier but are approximately linear over two decades in all but two cases. Using the simple linear models, we can conclude that the range of performance variation among all skiers in the sample is contained within the approximate limits ± 1.4 percent/year in the pace of the ski races, with the positive numbers indicating degrading performance and the negative numbers indicating improving performance. The trends are slow and steady, with no evidence for discontinuous performance changes that one might expect from a sudden change in health, dramatic improvement in ski equipment, changes in technique or motivation.

The simple mean for the entire sample is $+0.1 \pm 0.1$ (standard deviation of the mean)

percent/year change in performance. This is to be compared to the previous studies of aging mentioned in the introduction that were based on cross-sectional studies of +0.6 to +0.8 percent/year. In Fig. 5 I have shown the frequency distribution for the performance estimators from Table 1. Note that the values cluster around +0.2 indicating that the sample average is skewed to slightly lower values by the four extreme examples of improving performance. I have summarized the mean results from this study in Table 2, along with estimates of age-degraded performance in swimming (a similar skill sport using most muscle groups) from other studies using both longitudinal and cross-sectional information. Note that the skiers in this study well outperform those from swimming, consistent with the generalities cited above. Also shown is the result of the polynomial fit used by Heil (2010) to determine age-graded results for the Boulder Mountain Tour; this is equivalent to a cross-sectional study of hundreds of skiers during one particular race. His polynomial produces an estimate of age-degraded performance that is similar to the ones for swimming listed in Table 2 and to the studies mentioned earlier in this paper.

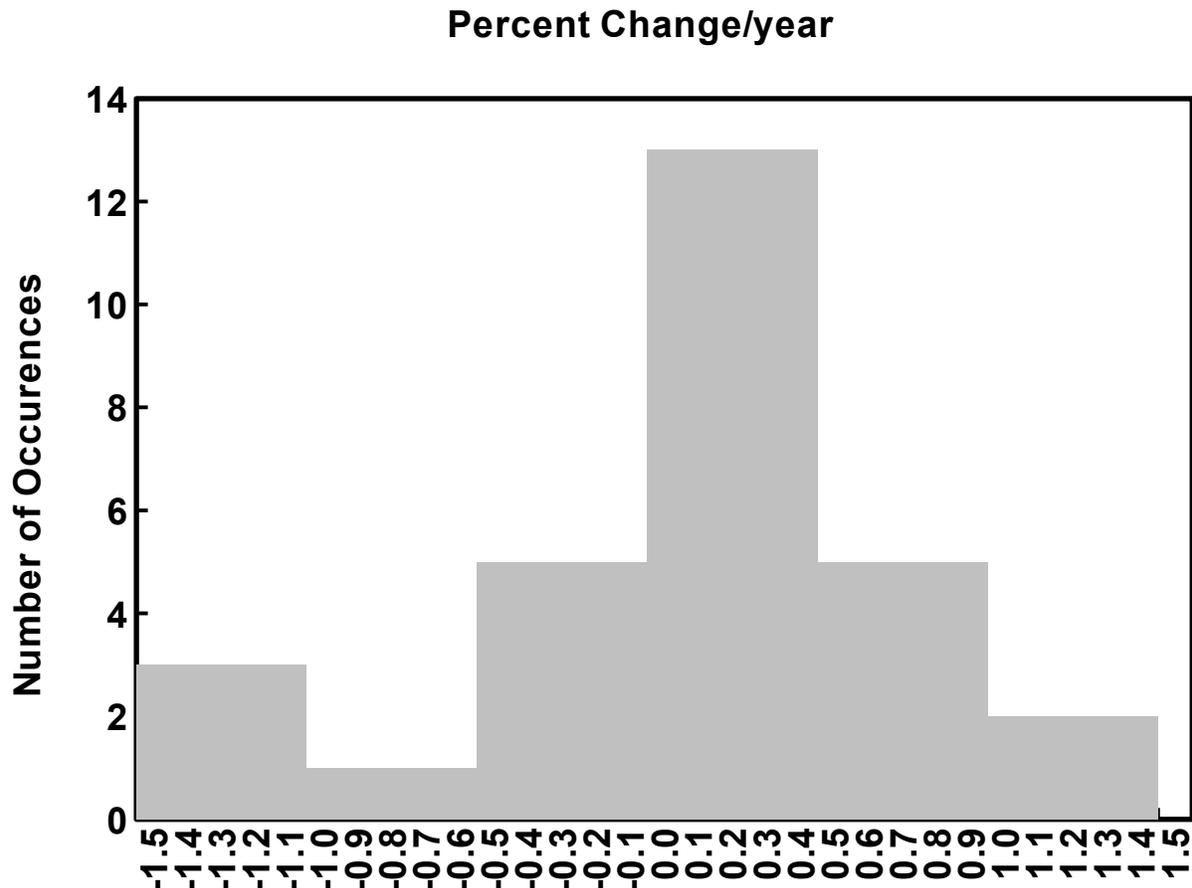


Figure 5. A frequency distribution of the performance slopes derived from the linear least-squares fits to the race pace data. All skier data are included in the figure.

Within the sample of 30 athletes, the 10 elite racers showed a mean change of $+0.16 \pm 0.17$ (s.d. mean) percent/year. The 20 citizen racers showed a mean of $+0.09 \pm 0.15$ (s.d. mean) percent/year. There is no statistically significant difference between the two groups. Separating genders I find that the women show a rate of change of $+0.07 \pm 0.19$ (s.d. mean) percent/year while the men were $+0.15 \pm 0.15$ (s.d. mean) percent/year. Again, there is no statistically significant difference between genders. Among the 10 elite racers, 5 showed improvement while there were 5 improvements out of 20 among the citizen racers. *This represents one-third of the entire population that improved over time-spans of order two decades.* Of the 10 athletes who improved, 3 were men and 7 were women.

It seemed to me at the outset that there should be no limit to how fast an athlete slows down with age. I am personally acquainted with older athletes who have heart problems that limit their performance, or joint issues or eyesight issues or complete lack of motivation. In the latter case there are some racers known to me who simply enter races “just to do them” but who have lost all interest in serious racing. If these skiers had been included in the study, it would move the frequency distribution shown in Fig. 5 to the right. I have tried to eliminate these but some may inevitably remain. So, looking at the slower end of the study is not particularly useful. However, it is of interest to look at the four most extreme cases of improving performance. We may try to assess whether or not these four are members of a different population. The mean rate of change for this group of four skiers (numbers 8,10,12 and 16) is -1.06 ± 0.17 (s.d. population). The mean rate of change for the remaining 26 skiers is $+0.29 \pm 0.48$ (s.d. population). We may use Student's t-test (Press, *et al.* 1987) to see if the means of the two populations are different. The result of the test is that the two group means are statistically different to a high level of significance ($p < 0.0001$).

We may ask why these four skiers differ so markedly from the rest of the group. One is tempted to explain the difference as due to improvements in ski equipment. After all newer materials are stronger and lighter resulting in more efficient performance for the skier. According to the manufacturers, the newer bases respond better to wax than the older ones and the newer waxes are generally faster overall. This hypothesis may be immediately rejected since this equipment is readily available to all skiers in the sample. Similarly one may question whether the difference arises because of differences in attention to technique or training method. Unfortunately, this is one of the uncontrolled parts of this study. Among the fastest-improving four skiers, two spend most of their training time on technique and short, high-intensity sessions and the other two on high-volume training. Two have coaches and two do not. All four train over 600 hours/year. Thus the reason for the difference between the fastest four and the others remains unknown.

The relationships developed in this study are likely to be the least optimistic estimates of age-related trends in performance in the sense that the scatter is maximized due to the uncontrolled factors mentioned in the introduction. The relationships can undoubtedly be improved with additional effort. For example, it was apparent to me during tabulation of race results that certain races produced systematically faster results, while others produced slower results, independent of variable weather conditions. These could be adjusted for by considering the mean behavior for athletes who overlap in these races or even by using the top 10 finishers (for example) in each race. Further, one could look at the change of ski pace versus race distance and apply additional corrections for race distance. If these kinds of

corrections actually reduced the scatter, it would be useful for trying to pick out trends over a shorter time baseline or for assessing more detailed models of age-related trends (that is, something other than linear change of performance).

B. Implications for aging and coaching

The longitudinal and cross-sectional studies referenced earlier concentrate on statistical samples of athletic populations that can be compared to the general population. This information is critical for health-care professionals who want scientific studies to be used as a basis for exercise recommendations made to aging adults (Elsawy & Higgins 2010; Nelson *et al* 2007). It is of further value to exercise physiologists who are interested in the acceleration of aging and its cause. Hence we see the large amounts of research devoted to the exact nature of the curves describing the aging process and, for example, the importance of including (or not) a quadratic term in the mathematical models (Young & Starkes 2005; Young, *et al* 2008). For the highly-motivated master's athlete, these statistical studies are not so satisfying. All predict that the aging athlete's performance will decline. The most optimistic outlook is presented in Young & Starkes (2005) in a study of runners. They state in their conclusion that "The moderation of age-related performance decline in the longitudinal sample [compared to the cross-sectional sample] was particularly evident for the 10 km event compared to the 1500 m distance." Nevertheless, it is still declining performance. Certainly, declining performance is not in question near the ultimate end of an athlete's life but is it necessarily the case throughout most of an athlete's competitive career? Examples of improvement in later-life performances are scarce in the scientific literature but a few have been emphasized in the popular press (Heine 2012; Mascarelli 2011).

One of the major conclusions from this study is that the aging athlete need not expect to show degrading performance with advancing age, at least in the sport of cross-country skiing. In fact, one out of three athletes in the present sample actually improve their ski performance consistently over two decades. The effect of age on the performance slope is not obvious. For example, of the four most extreme examples of improving performance, two of the skiers were over 70 while two were under 70 at the end of the study. Among the four most extreme examples of degrading performance, two skiers were over 70 and two were under 50 at the conclusion of the study. *There is no clear-cut indication that the older athletes necessarily show the fastest rate of decline.* Unfortunately, there is also no obvious characteristic that indicates which athlete will fall into which group. That is, there is no recipe for success (as measured by improving performance with advancing age) that can be gleaned from this study. Perhaps such a recipe could be derived from a more carefully regulated choice of athletes, including detailed interviews regarding health issues, training programs and motivation, for example. It would also be interesting to look for cases where a skier showing declining performance (at any age) has been able to reverse the trend. More effort is needed in skiing and other sports to identify those anomalous individuals who actually improve with age beyond what is normally considered the age of peak performance and to try to identify why they improve.

The generally slow trends evidenced in this study can present challenges to the master's athlete who may feel an inevitable limit to available time. After all, how many among us are willing to make a 20 year commitment to gain 20% in performance? My own personal

(anecdotal) experience with master's coaching in swimming is that most adults want sudden, short-term gains in performance and generally are not willing to be so dedicated. That is, they are looking for one "magic bullet" which will move them up a few places in the rankings. It is rare that someone who is 60 will develop a plan for themselves when they are, say, 80. The findings from this study suggest that one may need to have such a plan if winning or improvement is a big issue. For example, in 1991 skier number 8 skied at an average pace of 4.8 min/km while skier number 9 skied at an average pace of 3.8 min/km. If skier 9 wanted to beat skier number 8 *on the average*, she would have to wait until about 2010, 19 years later! That is not to say that number 9 would not have beaten number 8 at occasional races somewhere in between because 8 could have a statistically good race while 9 a statistically bad race, but not in the average.

My supposition is that most masters skiers who are not among the elite can improve through specific efforts in coached programs. In an article advocating for coached master's nordic ski programs, Engen (2003) states that "becoming a good nordic skier takes years of practice and specialized training." Ericsson (2003) discusses the development of expert skills in a wide variety of endeavors from sport to music. He identifies stages of development in the learner and concludes that the recreational participant reaches a level of skill that is considered to be acceptable and that further practice at this level can produce no additional improvement. For the experts, he notes that thousands of hours are required practicing alone trying various new and specific changes in order to reach the highest levels of expertise. This is certainly consistent with the slow changes for improvement noted in this study. The question remains whether an amateur athlete who expresses a desire to improve is really willing to commit the time and effort to the task.

Conclusion

In this paper I have presented results of ski race performance for a select group of citizen and elite nordic skiers. The results show that the general decline in performance for the group is slower than that reported for other sports. Most notable is that one-third of the sample population showed measurable improvement in performance over the average time-span from age 39 to age 60. For the individual athlete this is encouraging news because it indicates that the age at which inevitable performance decline has not yet been clearly defined, at least in this one particular sport. However, it is possible that an athlete whose performance is declining may have to make a commitment to the sport that reaches well beyond what is desirable or possible in order to reverse the trend.

Table 1: Summary of data for skiers used in this study.

Skier No.	Gender	E or C	ΔT (yrs)	No. Races	$(B/\langle P \rangle) \pm \sigma$ (%)
1	M	C	20	34	0.36 ± 0.34
2	M	C	20	29	0.07 ± 0.33
3	M	C	22	24	0.52 ± 0.32
4	M	C	22	29	1.34 ± 0.39
5	F	C	18	21	-0.37 ± 0.56
6	M	C	22	42	0.42 ± 0.21
7	F	C	16	17	0.80 ± 0.50
8	F	C	22	46	-0.85 ± 0.35
9	F	C	21	22	0.39 ± 0.42
10	F	C	22	31	-1.06 ± 0.38
11	M	C	22	26	0.26 ± 0.34
12	M	C	22	47	-1.27 ± 0.28
13	M	C	18	44	0.03 ± 0.40
14	M	C	22	32	0.89 ± 0.41
15	M	C	20	38	0.78 ± 0.38
16	M	C	22	35	-1.08 ± 0.39
17	M	C	19	34	0.03 ± 0.43
18	F	E	25	15	-0.26 ± 0.31
19	F	E	18	36	-0.32 ± 0.43
20	M	E	22	50	0.18 ± 0.28
21	F	E	28	38	-0.05 ± 0.23
22	M	E	22	34	-0.42 ± 0.31
23	F	E	22	37	-0.22 ± 0.20
24	F	E	21	26	1.38 ± 0.28
25	M	E	22	40	0.09 ± 0.49
26	M	C	18	45	0.10 ± 0.44
27	F	E	21	52	0.85 ± 0.34
28	M	C	21	46	0.23 ± 0.35
29	F	C	20	28	0.24 ± 0.27
30	F	E	22	35	0.37 ± 0.36

Table 2: Comparison of results.

Parameter	Average change of performance (percent/year) between ages 39 and 60						
	This Study	Heil (2010)		Bongard <i>et al</i> (2007)		Donato <i>et al</i> (2003)	
Sport	Skiing	Skiing		Swimming		Swimming	
Length	3-90 km	32 km		1 hour		1500 m	
Nature	Longitudinal	Cross-Sectional		Cross-Sectional		Longitudinal	
Gender	Both	Men	Women	Men	Women	Men	Women
Change	0.1±0.1	0.7	0.7	0.8	0.9	0.7	0.9

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